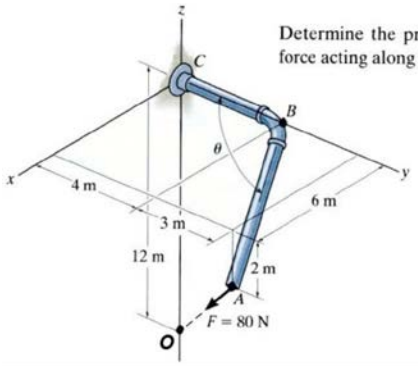
Çalışma Soruları



Determine the projected component of the 80-N force acting along the axis *AB* of the pipe.

(1) Find position vector from A to O

PT COORDINATES B(0,4,0), A(6,7,-2), O(0,0,-12)

$\vec{r}_{AO} = (0-6)\hat{i} + (0-7)\hat{j} + (-12-(-2))\hat{k}$

$\vec{r}_{AO} = -6\hat{i} - 7\hat{j} - 10\hat{k}$

(2) Find unit vector AO

$|r_{AO}| = \sqrt{6^2+7^2+10^2} = 13.6$

$\vec{u}_{AO} = \frac{\vec{r}_{FA}}{|r_{FA}|} = -\frac{6}{13.6}\hat{i} - \frac{7}{13.6}\hat{j} - \frac{10}{13.6}\hat{k}$

$\vec{u}_{AO} = -0.4412\hat{i} - 0.5147\hat{j} - 0.7353\hat{k}$

(3) Express 80 N force in cartesian vector format

$\vec{F} = (80\,N)(-0.4412\hat{i} - 0.5147\hat{j} - 0.7353\hat{k})$

$\vec{F} = \{-35.3\hat{i} - 41.2\hat{j} - 58.8\hat{k}\}\,N$

(4) Find position vector AB

$\vec{r}_{AB} = (0-6)\hat{i} + (4-7)\hat{j} + (0-(-2))\hat{k}$

$\vec{r}_{AB} = -6\hat{i} - 3\hat{j} + 2\hat{k}$

(5) Find unit vector AB

$|r_{AB}| = \sqrt{(6)^2+(-3)^2+(2)^2} = 7$

(5) Find unit vector AB

$|r_{AB}| = \sqrt{(6)^2+(-3)^2+(2)^2} = 7$

$\vec{u}_{AB} = -\frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k}$

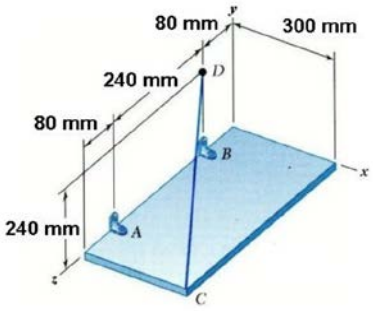
(6) Find the component of the 80 N force acting along AB.

Calculate the dot product $\vec{F}_{80N} \cdot \vec{U}_{AB}$

$F_{AB} = \vec{F} \cdot \vec{u}_{AB}$

$F_{AB} = (-35.3\hat{i} - 41.2\hat{j} - 58.8\hat{k}) \cdot (-\frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{2}{7}\hat{k})$

$F_{AB} = 30.3 + 17.7 - 16.8 = \boxed{31.1\,N}$



GIVEN: A shelf is supported by two brackets and a wire (CD). The tension in the wire is 200N.

FIND: The moment about hinge A of the force in wire CD.

A (0, 0, 320)
C (300, 0, 400)
D (0, 240, 80)

We want to find $\vec{M}_A = \vec{r} \times \vec{F}_{CD}$ **. REMEMBER, the position vector can intersect the line of force at any point, as long as it goes through point A.**

(1) Pick point C and find $\vec{r}_{AC}$.

$\vec{r}_{AC} = (300 - 0)\hat{i} + (400 - 320)\hat{k}$

$\vec{r}_{AC} = \{300\hat{i} + 80\hat{k}\}$ mm

(2) Calculate $\vec{F}_{CD}$ **using** $\vec{u}_{CD}$.

$\vec{r}_{CD} = (0-300)\hat{i} + (240-0)\hat{j} + (80-400)\hat{k}$

$\vec{r}_{CD} = \{-300\hat{i} + 240\hat{j} - 320\hat{k}\}$ mm $\Rightarrow |r_{CD}| = 500$

$\vec{u}_{CD} = -0.6\hat{i} + 0.48\hat{j} - 0.64\hat{k}$

$\vec{F}_{CD} = (200)(-0.6\hat{i} + 0.48\hat{j} - 0.64\hat{k})$ N

$\vec{F}_{CD} = \{-120\hat{i} + 96\hat{j} - 128\hat{k}\}$ N

(3) Calculate $\vec{M}_A$ **using a vector cross product.**

$\vec{M}_A = (300\hat{i} + 80\hat{k}) \times (-120\hat{i} + 96\hat{j} - 128\hat{k})$

$\vec{M}_A =$

$\hat{i}$	$\hat{j}$	$\hat{k}$	$\hat{i}$	$\hat{j}$
300	0	80	300	0
−120	96	−128	−120	96

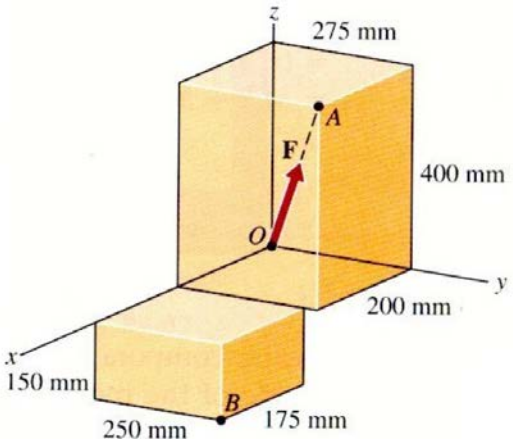
(− − − ; + + +)

$\vec{M}_A = (-120)(80)\hat{j} + (96)(300)\hat{k} - (96)(80)\hat{i} - (-128)(300)\hat{j}$

$\vec{M}_A = \{-7{,}680\hat{i} + 28{,}800\hat{j} + 28{,}800\hat{k}\}$ Nmm

$\vec{M}_A = \{-7.6\hat{i} + 28.8\hat{j} + 28.8\hat{k}\}$ Nm

A force with a magnitude of 840 N acts at a point in a body as shown.



Determine

a. The moment of the force about point *B*.

b. The direction angles associated with the unit vector along the axis of the moment.

c. The perpendicular distance *d* from point *B* to the line of action of the force.

(1) Express F as a Cartesian vector.

$\vec{r}_{OA} = \{200\hat{i} + 275\hat{j} + 400\hat{k}\}$ mm

$|\vec{r}_{OA}| = 525$

$\vec{u}_{OA} = \frac{200}{525}\hat{i} + \frac{275}{525}\hat{j} + \frac{400}{525}\hat{k}$

$\vec{F} = (840)\left(\frac{200}{525}\hat{i} + \frac{275}{525}\hat{j} + \frac{400}{525}\hat{k}\right)$

$\vec{F} = \{320\hat{i} + 440\hat{j} + 640\hat{k}\}$ N

(2) Pick a position vector to use as a moment arm.

$\vec{r}_{BO} = \{-375\hat{i} - 250\hat{j} + 150\hat{k}\}$ mm

$\vec{r}_{BA} = \{-175\hat{i} + 25\hat{j} + 550\hat{k}\}$ mm

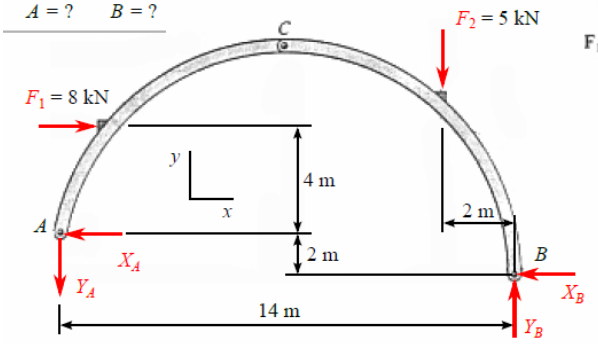
(3) Perform cross product operation to get $\vec{M}_B$

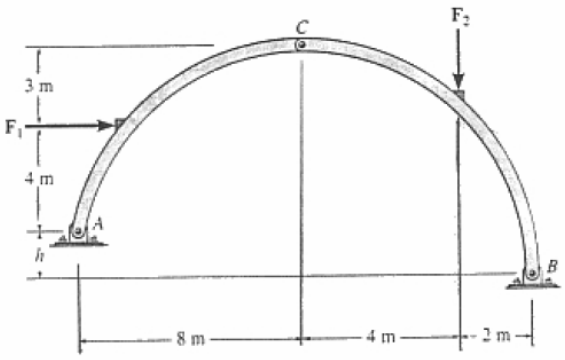
$$\vec{M}_B = \vec{r}_{BO} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -375 & -250 & 150 \\ 320 & 440 & 640 \end{vmatrix}$$

$\vec{M}_B = \{-226{,}000\hat{i} + 288{,}000\hat{j} - 85{,}000\hat{k}\}$ N mm

5. $F_1 = 8$ kN ve $F_2 = 5$ kN ise A ve B mesnetlerindeki tepkileri bulunuz. $h = 2$ m.

$A = ?$ $B = ?$



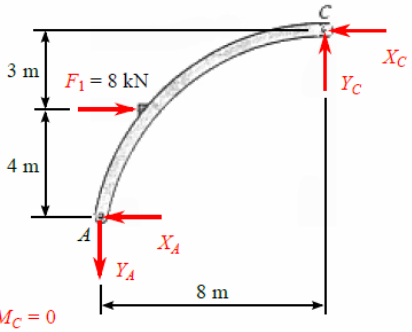


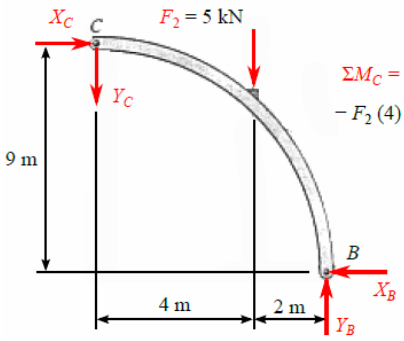
$\Sigma M_A = 0$

$-F_1(4) - F_2(12) - X_B(2) + Y_B(14) = 0$

$\Sigma M_B = 0$

$-F_1(6) + F_2(2) + X_A(2) + Y_A(14) = 0$





$\Sigma M_C = 0$

$-F_2(4) - X_B(9) + Y_B(6) = 0$

$X_B = 2.39$ kN

$Y_B = 6.91$ kN

$B^2 = X_B^2 + Y_B^2$

$B = 7.31$ kN

$\Sigma M_C = 0$

$F_1(3) - X_A(7) + Y_A(8) = 0$

$X_A = 5.61$ kN

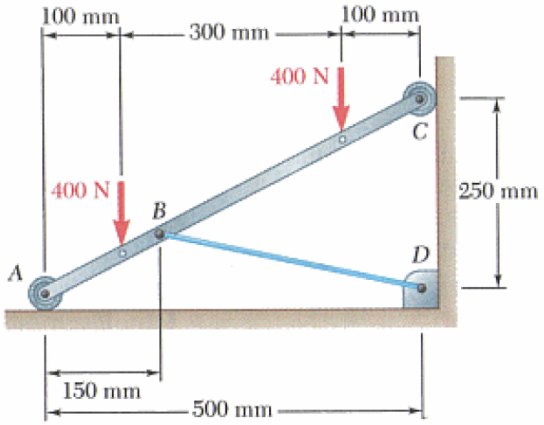
$Y_A = 1.91$ kN

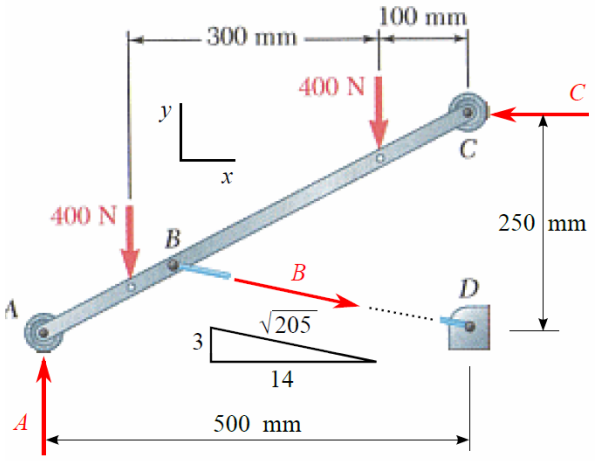
$A^2 = X_A^2 + Y_A^2$

$A = 5.93$ kN

- Ağırlığı ihmal edilen ve şekildeki gibi mesnetlenmiş ve yüklenmiş olan dengedeki çubuğun A, B ve C noktalarındaki pimlerin taşıdığı kuvvetlerin şiddetlerini hesaplayınız.

$A = ?$ $B = ?$ $C = ?$





$\Sigma F_x = 0 \rightarrow \quad B\dfrac{14}{\sqrt{205}} - C = 0$

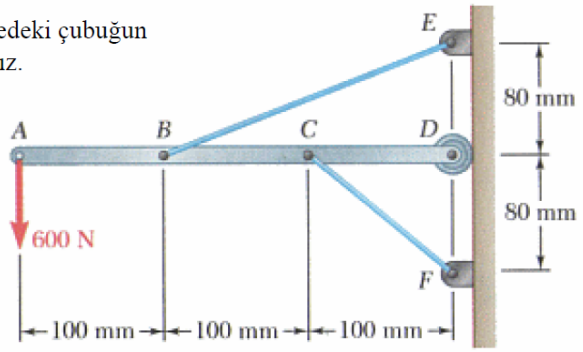
$\Sigma F_y = 0 \rightarrow \quad A - 400 - B\dfrac{3}{\sqrt{205}} - 400 = 0$

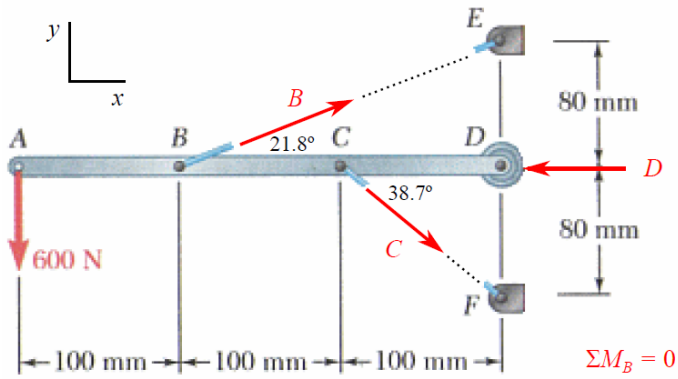
$\Sigma M_D = 0 \rightarrow \quad -A\,(500) + 400\,(400 + 100) + C\,(250) = 0$

$A = 1100$ N $\quad B = 1432$ N $\quad C = 1400$ N

- Ağırlığı ihmal edilen ve şekildeki gibi mesnetlenmiş ve yüklenmiş olan dengedeki çubuğun B, C ve D noktalarındaki pimlerin taşıdığı kuvvetlerin şiddetlerini hesaplayınız.

$B = ?$ $C = ?$ $D = ?$





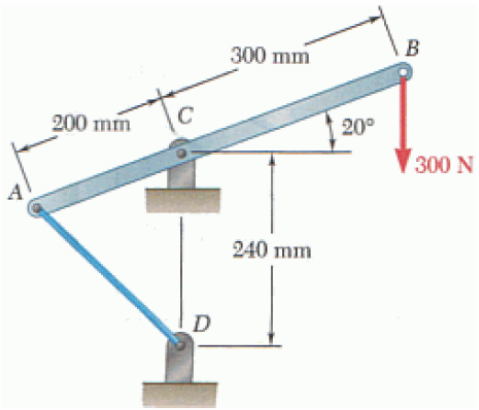
$\Sigma M_C = 0$

$600\,(200) - B\sin 21.8°\,(100) = 0$ $\quad B = 3231$ N

$\Sigma M_B = 0 \rightarrow \quad 600\,(100) - C\sin 38.7°\,(100) = 0$ $\quad C = 960$ N

$\Sigma F_x = 0 \rightarrow \quad B\cos 21.8° + C\cos 38.7° - D = 0$ $\quad D = 3750$ N

3. Şekildeki gibi yüklenmiş olan ACB çubuğu dengede olduğuna göre AD kablosundaki çekme kuvveti T yi ve C mesnedindeki tepki kuvvetinin şiddetini hesaplayınız.

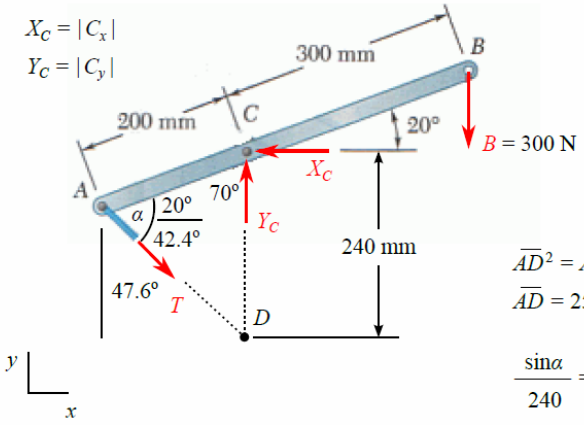


$B = 300$ N

$T = ?$

$C = ?$

Çubuğun kütlesi verilmediğine göre ağırlığını ihmal edeceğiz.



$$\overline{AD}^2 = \overline{AC}^2 + \overline{CD}^2 - 2\,\overline{AC}\,\overline{CD}\cos 70°$$

$$\overline{AD} = 254.25 \text{ mm}$$

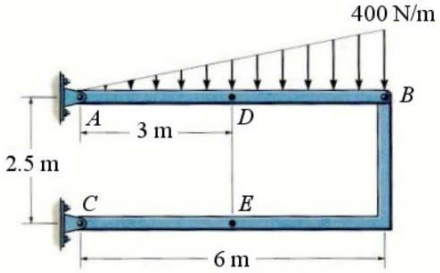
$$\frac{\sin\alpha}{240} = \frac{\sin 70°}{\overline{AD}} \rightarrow \alpha = 62.4°$$

$\Sigma M_C = 0 \rightarrow T\sin\alpha\,(200) - 300\,(300\cos 20°) = 0 \rightarrow T = 477$ N

$\Sigma F_x = 0 \rightarrow T\cos 42.4° - X_C = 0 \rightarrow X_C = 352$ N

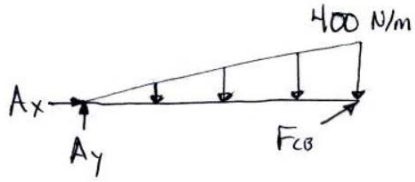
$\Sigma F_y = 0 \rightarrow -T\sin 42.4° + Y_C - 300 = 0 \rightarrow Y_C = 622$ N

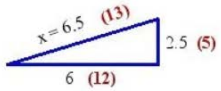
$C^2 = X_C^2 + Y_C^2 \rightarrow C = 715$ N



Determine the normal force, shear force and moment at a section passing through point D of the two-member frame.

(1) Draw a FBD of member AB and calculate the support reactions at A

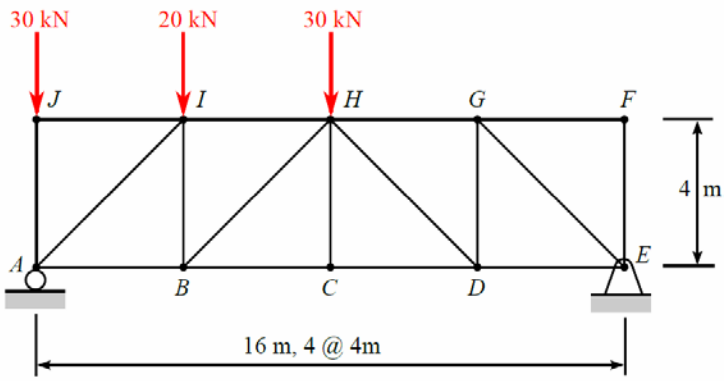




$+\uparrow \Sigma F_y: A_y + \left(\frac{5}{13}\right)(2080) - \left(\frac{1}{2}\right)(6)(400) = 0 \quad A_y = 400$ N

$+\rightarrow \Sigma F_x: A_x + \left(\frac{12}{13}\right)(2080) = 0 \quad A_x = -1920$ N

2. Şekildeki gibi mesnetlenmiş ve yüklenmiş olan kafes sistemde *GH*, *DH* ve *CD* çubuklarına gelen kuvvetleri bulunuz.

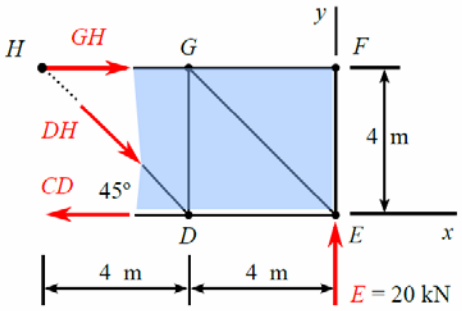


GH = ? *DH* = ? *CD* = ?

Kafes sistemin tamamının dengesinden:

$\Sigma M_A = 0 \rightarrow -20\,(4) - 30\,(8) + E\,(16) = 0$

$E = 20$ kN

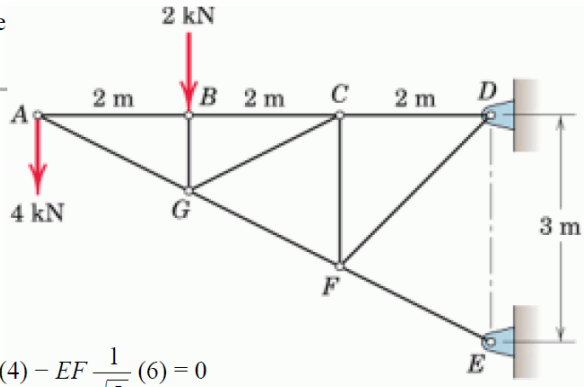


$\Sigma M_D = 0 \rightarrow 20\,(4) - GH\,(4) = 0 \rightarrow GH = 20$ kN

$\Sigma M_H = 0 \rightarrow 20\,(8) - CD\,(4) = 0 \rightarrow CD = 40$ kN

$\Sigma F_y = 0 \rightarrow 20 - DH \sin 45° = 0 \rightarrow DH = 14.14$ kN

2. Şekildeki gibi mesnetlenmiş ve yüklenmiş olan kafes sistemin *E* mesnedinde ortaya çıkan tepki kuvvetinin şiddetini bulunuz.

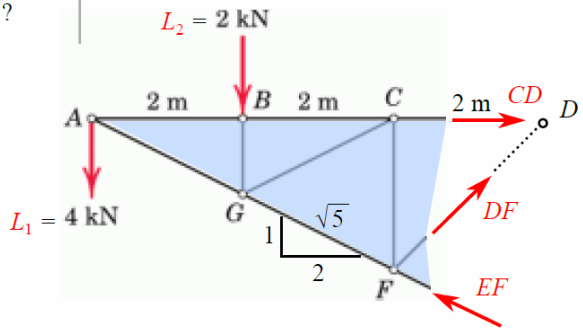


$L_1 = 4$ kN

$L_2 = 2$ kN

E = ?

E mesnedinde ortaya çıkan tepki kuvvetini doğuran sebep *EF* çubuğunun uyguladığı kuvvettir. Dolayısı ile mesnet tepkisi *EF* çubuğunun uyguladığı kuvvete eşittir. $E = EF$.



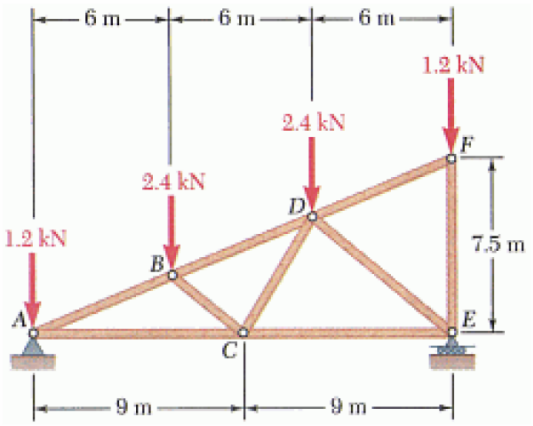
$\Sigma M_D = 0$

$4\,(6) + 2\,(4) - EF \dfrac{1}{\sqrt{5}}\,(6) = 0$

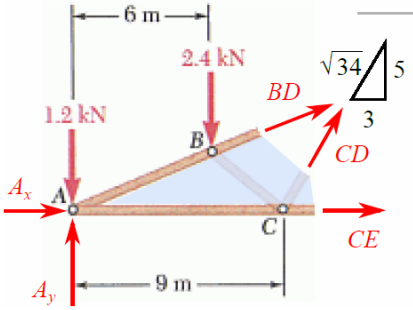
$EF = 11.93$ kN

$E = EF \rightarrow E = 11.93 \text{ kN} = \dfrac{16\sqrt{5}}{3}$ kN

- Şekildeki gibi mesnetlenmiş ve yüklenmiş olan kafes sistemin *CD* elemanının taşıdığı kuvveti hesaplayınız.



CD = ?

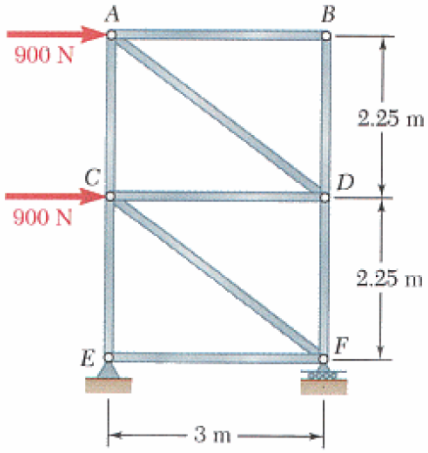


$\Sigma M_A = 0$

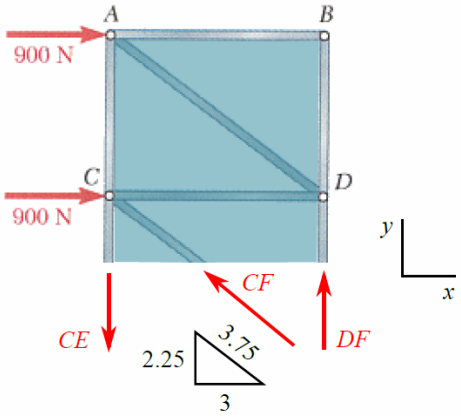
$$CD\,\frac{5}{\sqrt{34}}\,(9) - 2.4\,(6) = 0$$

$CD = 1.87$ kN

Şekildeki gibi mesnetlenmiş ve yüklenmiş olan kafes sistemin *CF* elemanına gelen kuvveti hesaplayınız.



CF = ?

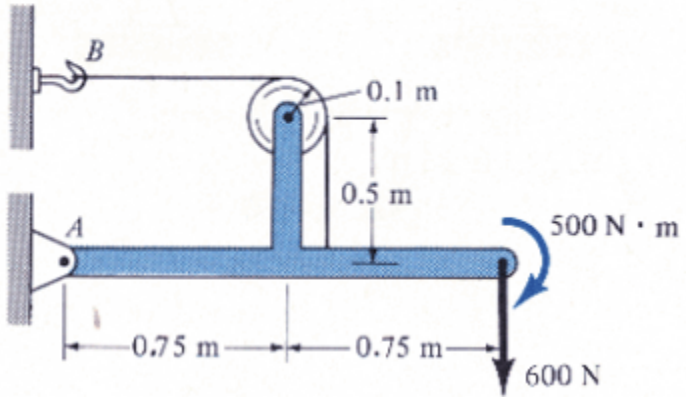


$\Sigma F_x = 0$

$$900 + 900 - CF\,\frac{3}{3.75} = 0$$

$CF = 2250$ N

6–59. Determine the horizontal and vertical components of force at pin *A*.



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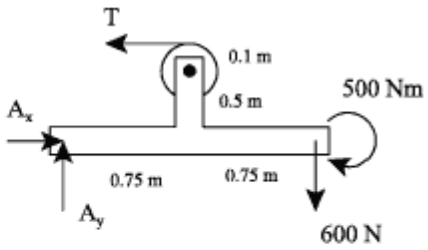
Determine the horizontal and vertical components of force at pin A.



We start with a FBD of the structure.

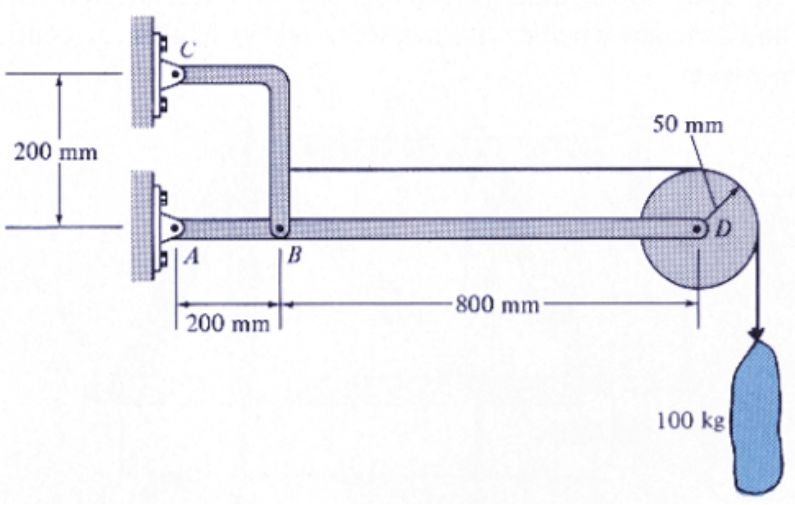
The equilibrium equations are:

$$\sum M_A: T(0.6m) - (600\,N)(1.5m) - 500\,Nm = 0 \;\Rightarrow\; T = 2330\,N$$

$$\sum F_x: -T + A_x = 0 \;\Rightarrow\; A_x = 2330\,N$$

$$\sum F_y: A_y - 600\,N = 0 \;\Rightarrow\; A_y = 600\,N$$

$$A_x = 2330\,N, \quad A_y = 600\,N$$



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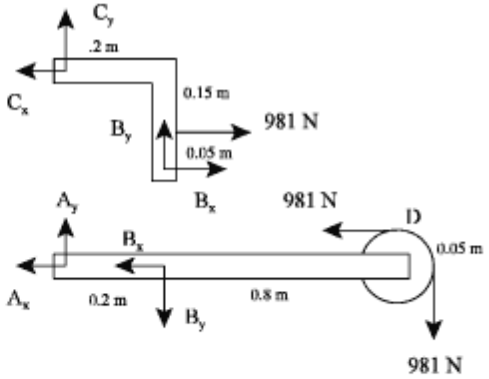
Determine the horizontal and vertical components of force that pins A, B, and C exert on their connecting members.

We start with a FBD of each member of the structure.



First using body ABD and then body BC we find the forces in the pin at B

$$\sum M_A: -B_y(0.2m) + (981N)(0.05m) - (981N)(1.05m) = 0 \Rightarrow B_y = -4905\,N$$

$$\sum M_C: (981N)(0.15m) + B_x(0.2m) + B_y(0.2m) = 0 \Rightarrow B_x = 4170\,N$$

Now we can work with body ABD to find the forces in the pin at A

$$\sum F_x: -A_x - B_x - 981N = 0 \Rightarrow A_x = -5150\,N$$

$$\sum F_y: A_y - B_y - 981N = 0 \Rightarrow A_y = -3920\,N$$

And finally working with body BC we find the forces in the pin at C

$$\sum F_x: -C_x + B_x + 981 = 0 \Rightarrow C_x = 5150\,N$$

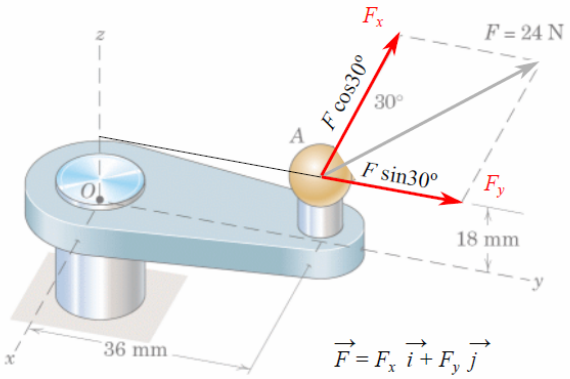
$$\sum F_y: C_y + B_y = 0 \Rightarrow C_y = 4905\,N$$

In Summary we have found that

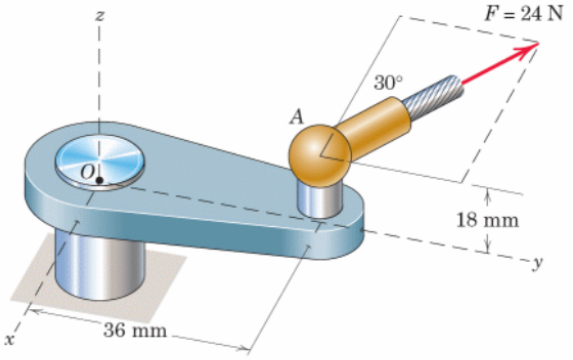
$$A_x = 5150\,N, \quad A_y = 3920\,N$$
$$B_x = 4170\,N, \quad B_y = 4910\,N$$
$$C_x = 5150\,N, \quad C_y = 4910\,N$$

1. Şekildeki krankın A noktasına bağlanmış olan bir kablonun A noktasına uyguladığı kuvvet 24 N dur. Bu kuvvetin O noktasına göre momentini birim vektörler cinsinden ifade ediniz.

$F = 24$ N $\quad \vec{M}_O = ?$



$\vec{F} = F_x\,\vec{i} + F_y\,\vec{j}$



Varignon teoreminden faydalanarak
O noktasından geçen eksenlere göre momentler yazılır.
Momentlerin yönü sağ el kuralı ile bulunur.

$M_{Ox} = M_x = -F\sin30°\,(18) = -216$ N·mm

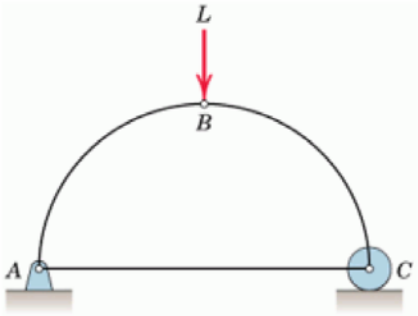
$M_{Oy} = M_y = -F\cos30°\,(18) = -374$ N·mm

$M_{Oz} = M_z = F\cos30°\,(36) = 748$ N·mm

$\vec{M}_O = M_{Ox}\,\vec{i} + M_{Oy}\,\vec{j} + M_{Oz}\,\vec{k} \quad \rightarrow \quad \vec{M}_O = -216\,\vec{i} - 374\,\vec{j} + 748\,\vec{k}$ N·mm

2. Şekildeki gibi mesnetlenmiş ve yüklenmiş olan kafes sistemin AC elemanının taşıdığı kuvvetin şiddetini bulunuz. AC ve BC elemanları çeyrek çember şeklindedir.

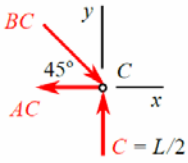
L $\quad AC = f(L) = ?$

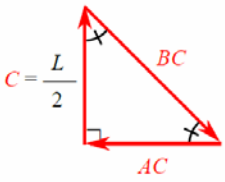


Kafes sistemin tamamının dengesinden:

$\Sigma M_A = 0 \quad \rightarrow \quad C\,(2\,r) - L\,(r) = 0 \quad \rightarrow \quad C = L/2$

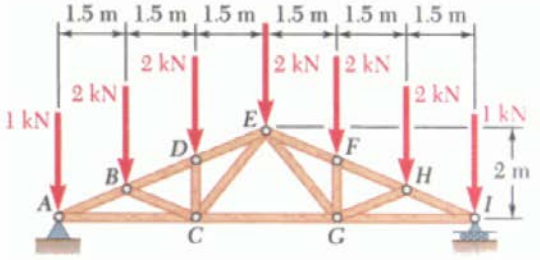
C düğümündeki pimin dengesinden:



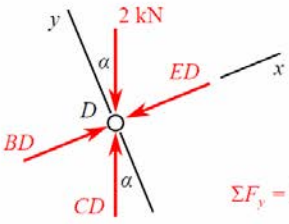


$AC = \dfrac{L}{2}$

2. Şekildeki gibi mesnetlenmiş ve yüklenmiş olan kafes sistemin CD elemanının taşıdığı kuvvetin şiddetini bulunuz.



$CD = ?$ $\quad$ D düğümündeki pimin dengesinden:



$\Sigma F_y = 0 \quad \rightarrow \quad CD\cos\alpha - 2\cos\alpha = 0$

$CD = 2$ kN