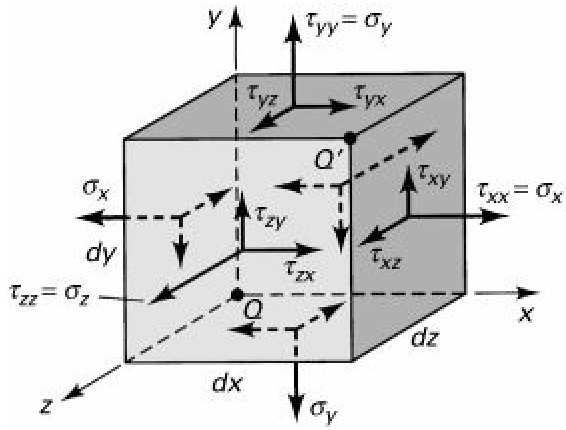
Preparation for the mid-term exam.

The positive directions of stresses..



The stress tensor:

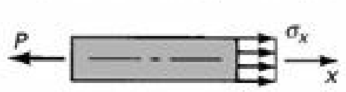
$$\begin{bmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{yx} & \sigma_y & \tau_{yz} \\ \tau_{zx} & \tau_{zy} & \sigma_z \end{bmatrix}$$

Triaxial stress:

$$\begin{bmatrix} \sigma_1 & 0 & 0 \\ 0 & \sigma_2 & 0 \\ 0 & 0 & \sigma_3 \end{bmatrix}$$

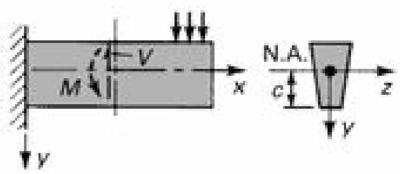
Plane stress:

$$\begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{bmatrix}$$



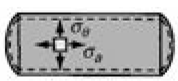
Axial loading: $\sigma_x = \frac{P}{A}$ (a)

Torsion: $\tau = \frac{T\rho}{J}$, $\tau_{max} = \frac{Tr}{J}$ (b)

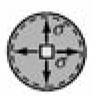


Bending: $\sigma_x = -\frac{My}{I}$, $\sigma_{max} = \frac{Mc}{I}$ (c)

Shear: $\tau_{xy} = \frac{VQ}{Ib}$ (d)

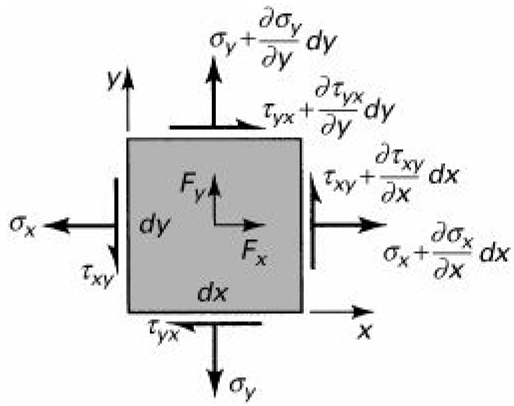


Cylinder: $\sigma_\theta = \frac{pr}{t}$, $\sigma_a = \frac{pr}{2t}$ (e)



Sphere: $\sigma = \frac{pr}{2t}$ (f)

Variation of stress within a body:



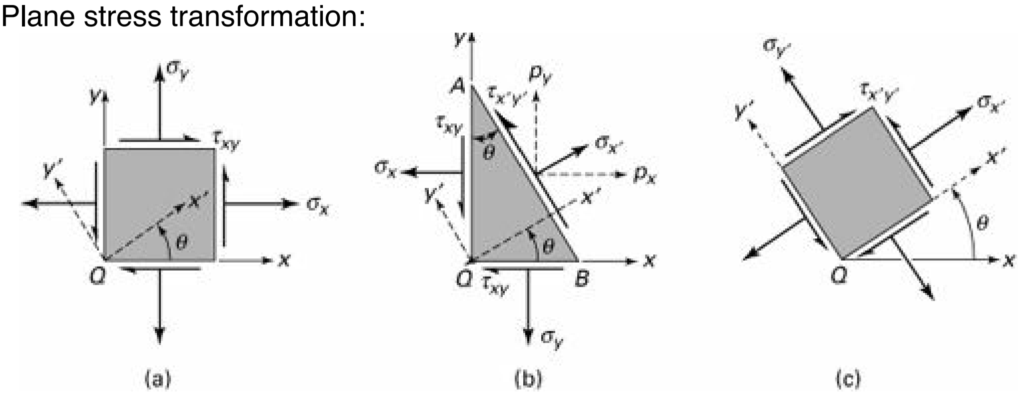
Differential equations of equilibrium for three dimensional stresses:

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} + F_x = 0$$

$$\frac{\partial \sigma_y}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yz}}{\partial z} + F_y = 0$$

$$\frac{\partial \sigma_z}{\partial z} + \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + F_z = 0$$

Plane stress transformation:



$$p_x = \sigma_x \cos\theta + \tau_{xy} \sin\theta \qquad \sigma_{x'} = p_x \cos\theta + p_y \sin\theta$$

$$p_y = \tau_{xy} \cos\theta + \sigma_y \sin\theta \qquad \tau_{x'y'} = p_y \cos\theta - p_x \sin\theta$$

$$\sigma_{x'} = \sigma_x \cos^2\theta + \sigma_y \sin^2\theta + 2\tau_{xy} \sin\theta\cos\theta$$

$$\tau_{x'y'} = \tau_{xy}(\cos^2\theta - \sin^2\theta) + (\sigma_y - \sigma_x)\sin\theta\cos\theta$$

Principal stresses for two dimensional problems:

$$\sigma_{\max,\min} = \sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

Planes of principal stresses:

$$\tan 2\theta_p = \frac{2\tau_{xy}}{\sigma_x - \sigma_y}$$

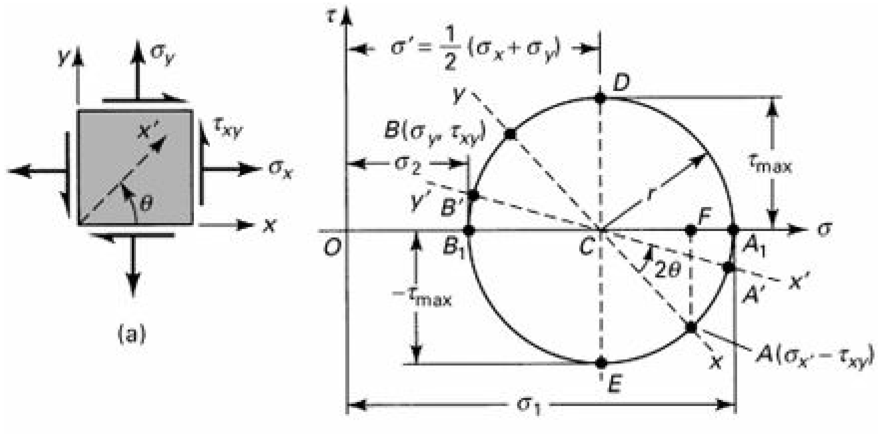
Maximum shearing stress:

$$\tau_{\max} = \pm\sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} = \pm\tfrac{1}{2}(\sigma_1 - \sigma_2)$$

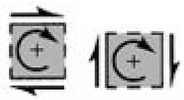
Planes of maximum shearing stress:

$$\tan 2\theta_s = -\frac{\sigma_x - \sigma_y}{2\tau_{xy}}$$

Mohr's circle for 2D case:

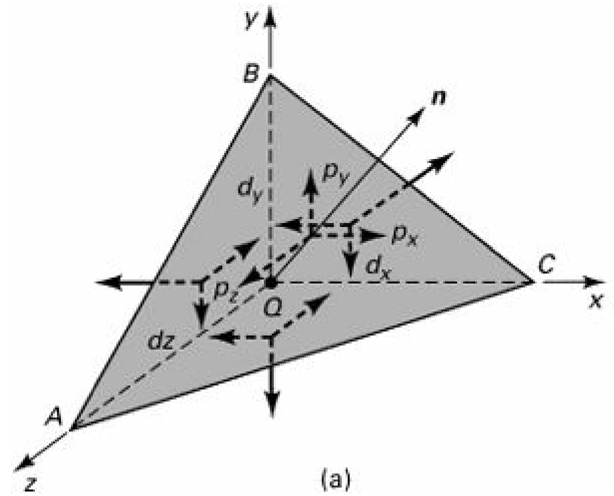


Sign convention for shearing stresses in Mohr's circle



Three dimensional stress transformation

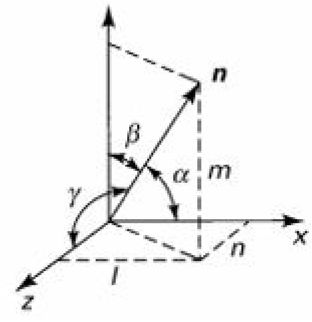
Stress components on a tetrahedron:



Areas of tetrahedron faces based on the area of ABC, indicated by A:

$$A_{QAB} = Al, \qquad A_{QAC} = Am, \qquad A_{QBC} = An$$

Direction cosines:



$$\cos\alpha = \cos(\boldsymbol{n}, x) = l$$
$$\cos\beta = \cos(\boldsymbol{n}, y) = m$$
$$\cos\gamma = \cos(\boldsymbol{n}, z) = n \qquad l^2 + m^2 + n^2 = 1$$

p is the stress resultant on the cut surface. px, py and pz are the cartesian components of p. Using force equilibrium in x, y and z directions the following relations are obtained:

$$p_x = \sigma_x l + \tau_{xy} m + \tau_{xz} n$$
$$p_y = \tau_{xy} l + \sigma_y m + \tau_{yz} n$$
$$p_z = \tau_{xz} l + \tau_{yz} m + \sigma_z n$$

Original coordinates: x, y, z . The rotated coordinates: x', y', z'.

The $x'y'z'$ and xyz systems are related by the direction cosines

Notation for direction cosines:

	x	y	z
x'	l_1	m_1	n_1
y'	l_2	m_2	n_2
z'	l_3	m_3	n_3

The normal stress in x direction after transformation:

$$\sigma_{x'} = p_x l_1 + p_y m_1 + p_z n_1$$

Direction cosines for stress transformation :

$$[T] = \begin{bmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{bmatrix}$$

Stress transformation:

$$[\sigma'] = [T][\sigma][T]^T$$

Principal stresses in 3D

$$(\sigma_x - \sigma_p)l + \tau_{xy} m + \tau_{xz} n = 0$$
$$\tau_{xy} l + (\sigma_y - \sigma_p)m + \tau_{yz} n = 0$$
$$\tau_{xz} l + \tau_{yz} m + (\sigma_z - \sigma_p)n = 0$$

$$\begin{vmatrix} \sigma_x - \sigma_p & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_p & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_p \end{vmatrix} = 0$$

$$\sigma_p^3 - I_1\sigma_p^2 + I_2\sigma_p - I_3 = 0$$

Stress invariants (Coordinate transformation does not change their values)

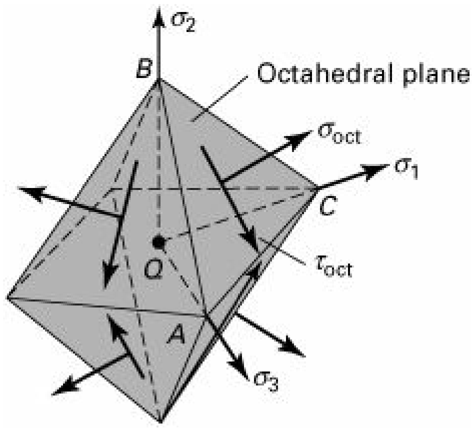
$$I_1 = \sigma_x + \sigma_y + \sigma_z$$

$$I_2 = \sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z - \tau_{xy}^2 - \tau_{yz}^2 - \tau_{xz}^2$$

$$I_3 = \begin{vmatrix} \sigma_x & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z \end{vmatrix}$$

$$I_1 = \sigma_{x'} + \sigma_{y'} + \sigma_{z'} = \sigma_x + \sigma_y + \sigma_z$$

Octahedral plane:



The normals of all faces make the same angle with x, y and z axes.

$$l = m = n = \frac{1}{\sqrt{3}}$$

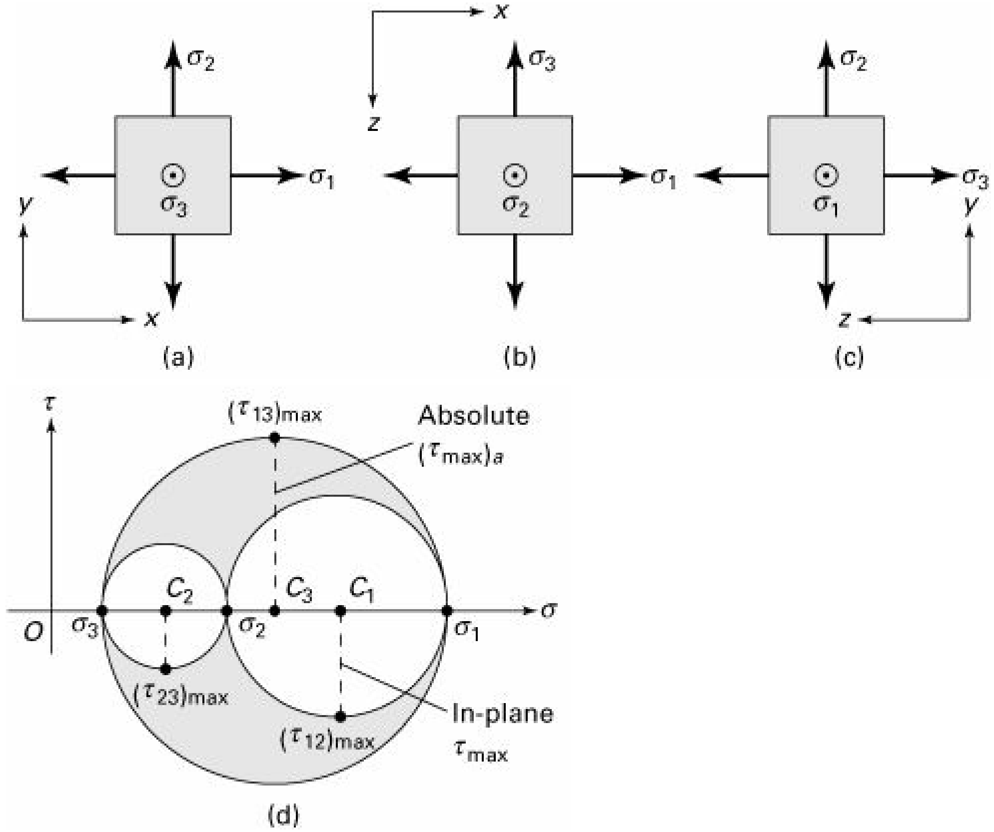
Octahedral normal stress:

$$\sigma_{oct} = \tfrac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$$

Octahedral shearing stress:

$$\tau_{oct} = \tfrac{1}{3}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{1/2}$$

Mohr’s circle for 3D

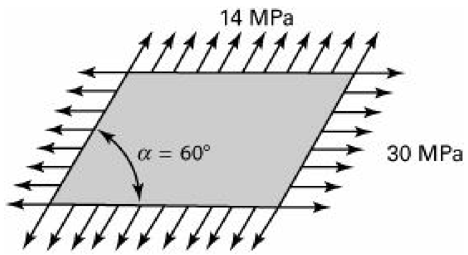


The shaded area in the figure represents the admissible state of stress.

Problems:

1.31. A thin skewed plate is subjected to a uniform distribution of stress along its sides, as shown in Fig. P1.31. Calculate (a) the stresses σ_x, σ_y, σ_{xy}, and (b) the principal stresses and their orientations.

Figure P1.31.

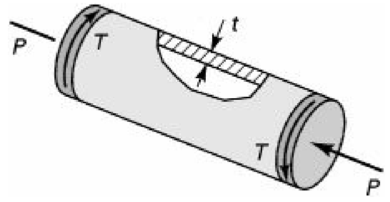


1.34. A thin-walled cylindrical tank of radius r is subjected simultaneously to internal pressure p and a compressive force P through rigid end plates. Determine the magnitude of force P to produce pure shear in the cylindrical wall.

Draw an element under pure shear a) Applying shear stresses b) Applying biaxial stress
Draw the Mohr’s circle for pure shear stress.

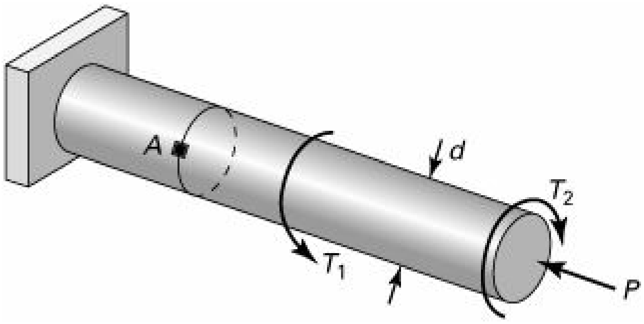
1.35. A thin-walled cylindrical pressure vessel of radius 120 mm and a wall thickness of 5 mm is subjected to an internal pressure of $p = 4$ MPa. In addition, an axial compression load of $P = 30\pi$ kN and a torque of $T = 10\pi$ kN · m are applied to the vessel through the rigid end plates (Fig. P1.35). Determine the maximum shearing stresses and associated normal stresses in the cylindrical wall. Show the results on a properly oriented element.

Figure P1.35.



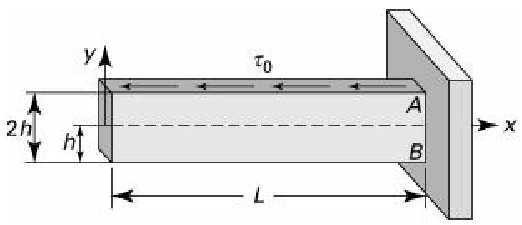
1.37. A shaft of diameter d carries an axial compressive load P and two torques T_1, T_2 (Fig. P1.37). Determine the maximum shear stress at a point A on the surface of the shaft. *Given*: $d = 100$ mm, $P = 400$ kN, $T_1 = 10$ kN · m, and $T_2 = 2$ kN · m.

Figure P1.37.



1.40. A cantilever beam of thickness t is subjected to a constant traction τ_0 (force per unit area) at its upper surface, as shown in Fig. P1.40. Determine, in terms of τ_0, h, and L, the principal stresses and the maximum shearing stress at the corner points A and B.

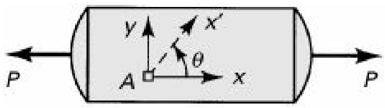
Figure P1.40.



1.47. The shearing stress at a point in a loaded structure is $\tau_{xy} = 40$ MPa. Also, it is known that the principal stresses at this point are $\sigma_1 = 40$ MPa and $\sigma_2 = -60$ MPa. Determine σ_x (compression) and σ_y and indicate the principal and maximum shearing stresses on an appropriate sketch.

1.53. A thin-walled cylindrical tank is subjected to an internal pressure p and uniform axial tensile load P (Fig. P1.53). The radius and thickness of the tank are $r = 0.45$ m and $t = 5$ mm. The normal stresses at a point A on the surface of the tank are restricted to $\sigma_{x'} = 84$ MPa and $\sigma_{y'} = 56$ MPa, while shearing stress $\tau_{x'y'}$ is not specified. Determine the values of p and P. Use $\theta = 30°$.

Figure P1.53.



1.59. The state of stress at a point in an x, y, z coordinate system is

$$\begin{bmatrix} 20 & 12 & -15 \\ 12 & 0 & 10 \\ -15 & 10 & 6 \end{bmatrix} \text{MPa}$$

Determine the stresses and stress invariants relative to the x', y', z' coordinate system defined by rotating x, y through an angle of 30° counterclockwise about the z axis.

1.65. At a point in a loaded structure, the stresses relative to an x, y, z coordinate system are given by

$$\begin{bmatrix} 30 & 0 & 20 \\ 0 & 0 & 0 \\ 20 & 0 & 0 \end{bmatrix} \text{MPa}$$

Determine by expanding the characteristic stress determinant: (a) the principal stresses; (b) the direction cosines of the maximum principal stress.

1.66. The stresses (in megapascals) with respect to an x, y, z coordinate system are described by

$$\sigma_x = x^2 + y, \qquad \sigma_z = -x + 6y + z$$
$$\sigma_y = y^2 - 5, \qquad \tau_{xy} = \tau_{xz} = \tau_{yz} = 0$$

At point (3, 1, 5), determine (a) the stress components with respect to x', y', z' if

$$l_1 = 1, \qquad m_2 = \tfrac{1}{2}, \qquad n_2 = \frac{\sqrt{3}}{2}, \qquad n_3 = \tfrac{1}{2}, \qquad m_3 = -\frac{\sqrt{3}}{2}$$

and (b) the stress components with respect to x'', y'', z'' if $l_1 = 2/\sqrt{5}$, $m_1 = -1/\sqrt{5}$, and $n_3 = 1$. Show that the quantities given by Eq. (1.34) are invariant under the transformations (a) and (b).

1.74. At a point in a loaded body, the stresses relative to an x, y, z coordinate system are

$$\begin{bmatrix} 40 & 40 & 30 \\ 40 & 20 & 0 \\ 30 & 0 & 20 \end{bmatrix} \text{MPa}$$

Determine the normal stress σ and the shearing stress τ on a plane whose outward normal is oriented at angles of 40°, 75°, and 54° with the x, y, and z axes, respectively.

1.77. The state of stress at a point in a member relative to an *x, y, z* coordinate system is given by

$$\begin{bmatrix} -100 & 0 & -80 \\ 0 & 20 & 0 \\ -80 & 0 & 20 \end{bmatrix} \text{MPa}$$

Calculate (a) the principal stresses by expansion of the characteristic stress determinant; (b) the octahedral stresses and the maximum shearing stress.

The example problems in the book should also be added this list of problems.

Strain in 3D

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial v}{\partial y}, \qquad \varepsilon_z = \frac{\partial w}{\partial z}$$

$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \qquad \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}, \qquad \gamma_{zx} = \frac{\partial w}{\partial x} + \frac{\partial u}{\partial z}$$

$$\gamma_{xy} = \gamma_{yx}, \qquad \gamma_{yz} = \gamma_{zy}, \qquad \gamma_{zx} = \gamma_{xz}$$

$$\varepsilon_{xy} = \tfrac{1}{2}\gamma_{xy}, \qquad \varepsilon_{yz} = \tfrac{1}{2}\gamma_{yz}, \qquad \varepsilon_{xz} = \tfrac{1}{2}\gamma_{xz}$$

Strain tensor in 3D

$$[\varepsilon_{ij}] = \begin{bmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{yx} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{zx} & \frac{1}{2}\gamma_{zy} & \varepsilon_z \end{bmatrix}$$

Equations of compatibility

$$\frac{\partial^2 \varepsilon_x}{\partial y^2} + \frac{\partial^2 \varepsilon_y}{\partial x^2} = \frac{\partial^2 \gamma_{xy}}{\partial x\, \partial y}, \qquad 2\frac{\partial^2 \varepsilon_x}{\partial y\, \partial z} = \frac{\partial}{\partial x}\left(-\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \varepsilon_y}{\partial z^2} + \frac{\partial^2 \varepsilon_z}{\partial y^2} = \frac{\partial^2 \gamma_{yz}}{\partial y\, \partial z}, \qquad 2\frac{\partial^2 \varepsilon_y}{\partial z\, \partial x} = \frac{\partial}{\partial y}\left(\frac{\partial \gamma_{yz}}{\partial x} - \frac{\partial \gamma_{xz}}{\partial y} + \frac{\partial \gamma_{xy}}{\partial z}\right)$$

$$\frac{\partial^2 \varepsilon_z}{\partial x^2} + \frac{\partial^2 \varepsilon_x}{\partial z^2} = \frac{\partial^2 \gamma_{xz}}{\partial z\, \partial x}, \qquad 2\frac{\partial^2 \varepsilon_z}{\partial x\, \partial y} = \frac{\partial}{\partial z}\left(\frac{\partial \gamma_{yz}}{\partial x} + \frac{\partial \gamma_{xz}}{\partial y} - \frac{\partial \gamma_{xy}}{\partial z}\right)$$

Strain transformation in 2D

$$\varepsilon_{x'} = \varepsilon_x \cos^2\theta + \varepsilon_y \sin^2\theta + \gamma_{xy}\sin\theta\cos\theta$$

$$\varepsilon_{x'} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\gamma_{x'y'} = -(\varepsilon_x - \varepsilon_y)\sin 2\theta + \gamma_{xy}\cos 2\theta$$

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

Principal strain directions:

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y}$$

Maximum shearing strains:

$$\gamma_{\max} = \pm 2\sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = \pm(\varepsilon_1 - \varepsilon_2)$$

Strain transformation in 3D

$$\varepsilon_{x'} = \varepsilon_x l_1^2 + \varepsilon_y m_1^2 + \varepsilon_z n_1^2 + \gamma_{xy} l_1 m_1 + \gamma_{yz} m_1 n_1 + \gamma_{xz} l_1 n_1$$

Principal strains in 3D

$$\varepsilon_p^3 - J_1 \varepsilon_p^2 + J_2 \varepsilon_p - J_3 = 0$$

Strain invariants:

$$J_1 = \varepsilon_x + \varepsilon_y + \varepsilon_z$$

$$J_2 = \varepsilon_x\varepsilon_y + \varepsilon_x\varepsilon_z + \varepsilon_y\varepsilon_z - \tfrac{1}{4}(\gamma_{xy}^2 + \gamma_{yz}^2 + \gamma_{xz}^2)$$

$$J_3 = \begin{vmatrix} \varepsilon_x & \frac{1}{2}\gamma_{xy} & \frac{1}{2}\gamma_{xz} \\ \frac{1}{2}\gamma_{xy} & \varepsilon_y & \frac{1}{2}\gamma_{yz} \\ \frac{1}{2}\gamma_{xz} & \frac{1}{2}\gamma_{yz} & \varepsilon_z \end{vmatrix}$$

Hooke's law

$$\sigma_x = E\varepsilon_x \quad \tau_{xy} = G\gamma_{xy}$$

Poisson's effect

$$\varepsilon_y = \varepsilon_z = -\nu\frac{\sigma_x}{E} \qquad \nu = -\frac{\text{lateral strain}}{\text{axial strain}}$$

Unit volume change (dilatations)

$$e = \frac{\Delta V}{V_o} = (1 - 2\nu)\varepsilon_x = \frac{1 - 2\nu}{E}\sigma_x$$

Generalized Hooke's law for homogenous elastic material:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix}$$

Stress-Strain relations

$$\varepsilon_x = \frac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)], \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_y = \frac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)], \qquad \gamma_{yz} = \frac{\tau_{yz}}{G}$$

$$\varepsilon_z = \frac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)], \qquad \gamma_{xz} = \frac{\tau_{xz}}{G}$$

Lamé constants

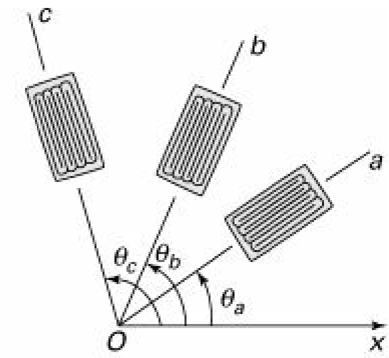
$$G = \frac{E}{2(1+\nu)} \qquad \lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}$$

Bulk modulus

$$K = -\frac{p}{e} = \frac{E}{3(1-2\nu)}$$

Here, p is hydrostatic pressure.

Strain rosette



$$\varepsilon_a = \varepsilon_x \cos^2\theta_a + \varepsilon_y \sin^2\theta_a + \gamma_{xy}\sin\theta_a\cos\theta_a$$
$$\varepsilon_b = \varepsilon_x \cos^2\theta_b + \varepsilon_y \sin^2\theta_b + \gamma_{xy}\sin\theta_b\cos\theta_b$$
$$\varepsilon_c = \varepsilon_x \cos^2\theta_c + \varepsilon_y \sin^2\theta_c + \gamma_{xy}\sin\theta_c\cos\theta_c$$

Strain energy density (strain energy for unit volume) for uniaxial case:

$$U_o = \int_0^{\varepsilon_x} \sigma_x \, d\varepsilon_x = \int_0^{\varepsilon_x} E\varepsilon_x \, d\varepsilon_x$$

Total strain energy for triaxial stress state:

$$U_o = \tfrac{1}{2}(\sigma_x \varepsilon_x + \sigma_y \varepsilon_y + \sigma_z \varepsilon_z)$$

Strain energy density for pure shear:

$$U_o = \tfrac{1}{2}\tau_{xy}\gamma_{xy} = \frac{1}{2G}\tau_{xy}^2 = \tfrac{1}{2}G\gamma_{xy}^2$$

$$U_o = \tfrac{1}{2}(\tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz})$$

Strain energy density for 3D stress state:

$$U_o = \tfrac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z + \tau_{xy}\gamma_{xy} + \tau_{yz}\gamma_{yz} + \tau_{xz}\gamma_{xz})$$

Strain energy for an axially loaded bar:

$$U = \int_V \frac{\sigma_x^2}{2E} dV = \int_0^L \frac{P^2}{2AE} dx$$

if bar is prismatic,

$$U = \frac{P^2L}{2AE}$$

Strain energy due to torsion for a circular shaft:

$$U = \int_0^L \frac{T^2}{2JG}\,dx$$

if the shaft is prismatic,

$$U = \frac{T^2L}{2JG}$$

Strain energy for beams in bending

$$U = \int_0^L \frac{M^2}{2EI}dx$$

Dilatational stress tensor

$$\begin{bmatrix} \sigma_m & 0 & 0 \\ 0 & \sigma_m & 0 \\ 0 & 0 & \sigma_m \end{bmatrix}$$

Here, $\sigma_m = \frac{1}{3}(\sigma_x + \sigma_y + \sigma_z)$

Dilatational strain energy density:

$$U_{ov} = \tfrac{3}{2}\sigma_m\,\varepsilon_m$$

Here, $\varepsilon_m = \frac{1}{3}(\varepsilon_x + \varepsilon_y + \varepsilon_z)$

Distortional stress tensor (deviator):

$$\begin{bmatrix} \sigma_x - \sigma_m & \tau_{xy} & \tau_{xz} \\ \tau_{xy} & \sigma_y - \sigma_m & \tau_{yz} \\ \tau_{xz} & \tau_{yz} & \sigma_z - \sigma_m \end{bmatrix}$$

Distortional strain energy:

$$U_{od} = \frac{3}{4G}\,\tau_{\text{oct}}^2$$

Strain energy

$$U_o = U_{ov} + U_{od}$$

Saint-Venant’s Principle

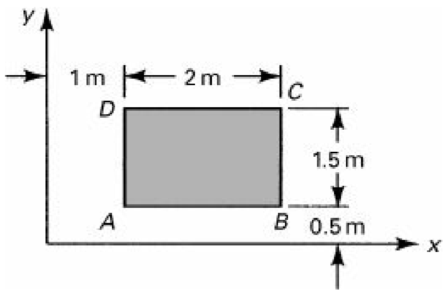
If actual distribution of forces is replaced by a statically equivalent system, the distribution of stress and strain throughout the body is altered only near the regions of load application.

Problems

2.2. Rectangle *ABCD* is scribed on the surface of a member prior to loading (Fig. P2.2). Following the application of the load, the displacement field is expressed by

$$u = c(2x + y^2), \qquad v = c(x^2 - 3y^2)$$

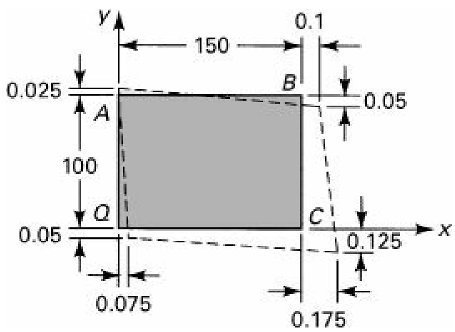
Figure P2.2.



where $c = 10^{-4}$. Subsequent to the loading, determine (a) the length of the sides *AB* and *AD*; (b) the change in the angle between sides *AB* and *AD*; and (c) the coordinates of point *A*.

2.9. A 100-mm by 150-mm rectangular plate *QABC* is deformed into the shape shown by the dashed lines in Fig. P2.9. All dimensions shown in the figure are in millimeters. Determine at point *Q* (a) the strain components ε_x, ε_y, γ_{xy}, and (b) the principal strains and the direction of the principal axes.

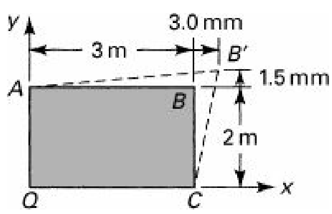
Figure P2.9.



2.15. The principal strains at a point are $\varepsilon_1 = 400\ \mu$ and $\varepsilon_2 = 200\ \mu$. Determine (a) the maximum shear strain and the direction along which it occurs and (b) the strains in the directions at $\theta = 30°$ from the principal axes. Solve the problem by using the formulas developed and check the results by employing Mohr's circle.

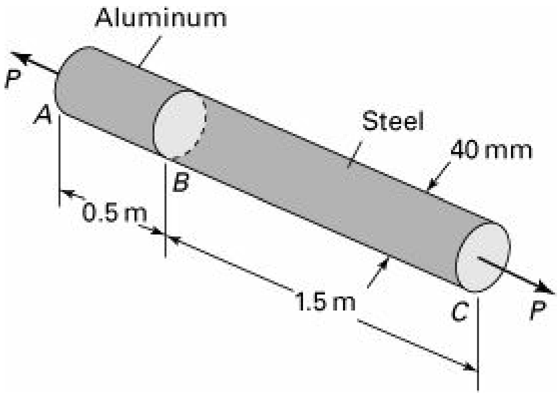
2.16. A 3-m by 2-m rectangular thin plate is deformed by the movement of point *B* to *B′* as shown by the dashed lines in Fig. P2.16. Assuming a displacement field of the form $u = c_1xy$ and $v = c_2xy$, wherein c_1 and c_2 are constants, determine (a) expressions for displacements *u* and *v*; (b) strain components ε_x, ε_y, and γ_{xy} at point *B*; and (c) the normal strain $\varepsilon_{x'}$ in the direction of line *QB*. Verify that the strain field is possible.

Figure P2.16.



2.27. A 40-mm diameter bar *ABC* is composed of an aluminum part *AB* and a steel part *BC* (Fig. P2.27). After axial force *P* is applied, a strain gage attached to the steel measures normal strain at the longitudinal direction as $\varepsilon_s = 600\ \mu$. Determine (a) the magnitude of the applied force *P*; (b) the total elongation of the bar if each material behaves elastically. Take $E_a = 70$ GPa and $E_s = 210$ GPa.

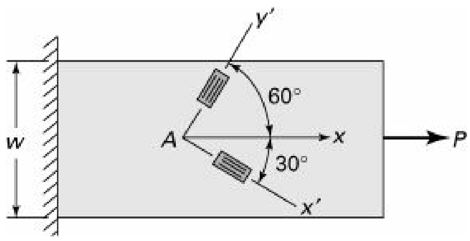
Figure P2.27.



2.29. The cast-iron pipe of length *L*, outer diameter *D*, and thickness *t* is subjected to an axial compressive *P*. Calculate (a) the change in length ΔL; (b) the change in outer diameter *D*; (c) the change in thickness Δt. *Given:* $D = 100$ mm, $t = 10$ mm, $L = 0.4$ m, $P = 150$ kN, $E = 70$ GPa, and $\nu = 0.3$.

2.34. A metallic plate of width *w* and thickness *t* is subjected to a uniform axial force *P* as shown in Fig. P2.34. Two strain gages placed at point *A* measure the strains $\varepsilon_{x'}$ and at 30° and 60°, respectively, to the axis of the plate. Calculate (a) the normal strains ε_x and ε_y; (b) the normal strains $\varepsilon_{x'}$ and $\varepsilon_{y'}$; (c) the shearing strain $\gamma_{x'y'}$. *Given:* $w = 60$ mm, $t = 6$ mm, $E = 200$ GPa, $\nu = 0.3$, and $P = 25$ kN.

Figure P2.34.



2.41. For a given steel, $E = 200$ GPa and $G = 80$ GPa. If the state of strain at a point within this material is given by

$$\begin{bmatrix} 200 & 100 & 0 \\ 100 & 300 & 400 \\ 0 & 400 & 0 \end{bmatrix} \mu$$

ascertain the corresponding components of the stress tensor.

2.42. For a material with $G = 80$ GPa and $E = 200$ GPa, determine the strain tensor for a state of stress given by

$$\begin{bmatrix} 20 & -4 & 5 \\ -4 & 0 & 10 \\ 5 & 10 & 15 \end{bmatrix} \text{MPa}$$

2.50. The stress field in an elastic body is given by

$$\begin{bmatrix} cy^2 & 0 \\ 0 & -cx^2 \end{bmatrix}$$

where c is a constant. Derive expressions for the displacement components $u(x, y)$ and $v(x, y)$ in the body.

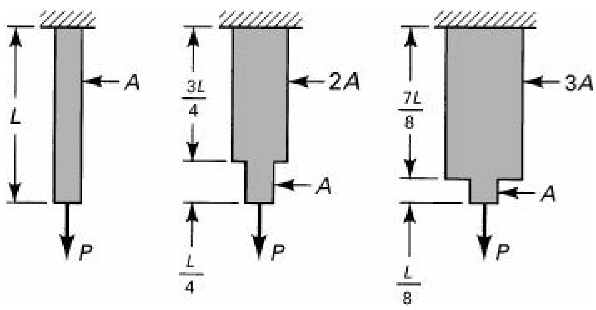
2.54. A bar of uniform cross-sectional area A, modulus of elasticity E, and length L is fixed at its right end and subjected to axial forces P_1 and P_2 at its free end. Verify that the total strain energy stored in the bar is given by

$$U = \frac{P_1^2 L}{2AE} + \frac{P_2^2 L}{2AE} + \frac{P_1 P_2 L}{AE} \tag{P2.54}$$

Note that U is *not* the sum of the strain energies due to P_1 and P_2 acting separately. Find the components of the energy for $P_1 = P_2 = P$ and $\nu = 0.25$.

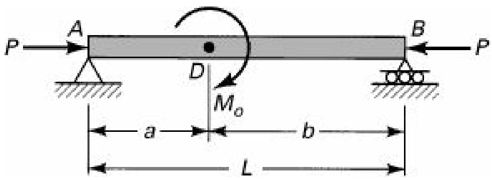
2.55. Three bars of successively larger volume are to support the same load P (Fig. P2.55). Note that the first bar has a uniform cross-sectional area A over its length L. Neglecting stress concentrations, compare the strain energy stored in the three bars.

Figure P2.55.



2.59. (a) Taking into account only the effect of normal stress, determine the strain energy of prismatic beam *AB* due to the axial force *P* and moment M_o acting simultaneously (Fig. P2.59). (b) Evaluate the strain energy for the case in which the beam is rectangular, 100-mm deep by 75-mm wide, $P = 8$ kN, $M_o = 2$ kN · m, $L = 1.2$ m, $a = 0.3$ m, $b = 0.9$ m, and $E = 70$ GPa.

Figure P2.59.



2.61. (a) Determine the strain energy of a solid brass circular shaft *ABC* loaded as shown in Fig. P2.61, assuming that the stress concentrations may be omitted. (b) Calculate the strain energy for $T = 1.4$ kN · m, $a = 500$ mm, $d = 20$ mm, and $G = 42$ GPa.

Figure P2.61.

2.66. The state of stress at a point is

$$\begin{bmatrix} 200 & 20 & 10 \\ 20 & -50 & 0 \\ 10 & 0 & 40 \end{bmatrix} \text{MPa}$$

Decompose this array into a set of dilatational stresses and a set of deviator stresses. Determine the values of *principal deviator stress*.

2.67. Calculate the strain energy per unit volume in changing the volume and in changing the shape of the material at any point on the surface of a steel shaft 120 mm in diameter subjected to torques of 20 kN · m and moments of 15 kN · m at its ends. Use $E = 200$ GPa and $\nu = 0.25$.

The example problems in the book should also be added this list of problems.